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10MAT41

Fourth Semester B.E. Degree Examination, June/July 2014

Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Obtain a solution upto the third approximation of y for x = 0.2 by Picard's method, given that $\frac{dy}{dx} + y = e^x$; $y(0) = 1$. **(06 Marks)**

b. Apply Runge-Kutta method of order 4, to find an approximate value of y for x = 0.2 in steps of 0.1, if $\frac{dy}{dx} = x + y^2$ given that y = 1 when x = 0. **(07 Marks)**

c. Using Adams-Bashforth formulae, determine y(0.4) given the differential equation $\frac{dy}{dx} = \frac{1}{2}xy$ and the data, y(0) = 1, y(0.1) = 1.0025, y(0.2) = 1.0101, y(0.3) = 1.0228. Apply the corrector formula twice. **(07 Marks)**

2 a. Apply Picard's method to find the second approximation to the values of 'y' and 'z' given that $\frac{dy}{dx} = z$, $\frac{dz}{dx} = x^3(y+z)$, given $y = 1$, $z = \frac{1}{2}$ when x = 0. **(06 Marks)**

b. Using Runge-Kutta method, solve $\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 + y^2 = 0$ for x = 0.2 correct to four decimal places. Initial conditions are x = 0, y = 1, y′ = 0. **(07 Marks)**

c. Obtain the solution of the equation $\frac{2d^2y}{dx^2} = 4x + \frac{dy}{dx}$ at the point x = 1.4 by applying Milne's method given that y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649, y(1.3) = 2.7514, y′(1) = 2, y′(1.1) = 2.3178, y′(1.2) = 2.6725 and y′(1.3) = 3.0657. **(07 Marks)**

3 a. Define an analytic function in a region R and show that f(z) is constant, if f(z) is an analytic function with constant modulus. **(06 Marks)**

b. Prove that $u = x^2 - y^2$ and $v = \frac{y}{x^2 + y^2}$ are harmonic functions of (x, y) but are not harmonic conjugate. **(07 Marks)**

c. Determine the analytic function f(z) = u + iv, if $u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$ and $f(\pi/2) = 0$. **(07 Marks)**

4 a. Find the images of the circles |z| = 1 and |z| = 2 under the conformal transformation $w = z + \frac{1}{z}$ and sketch the region. **(06 Marks)**

b. Find the bilinear transformation that transforms the points 0, i, ∞ onto the points 1, −i, −1 respectively. **(07 Marks)**

c. State and prove Cauchy's integral formula and hence generalized Cauchy's integral formula. **(07 Marks)**

PART – B

5 a. Obtain the solution of the equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(x^2-\frac{1}{4}\right)y=0$. **(06 Marks)**

b. Obtain the series solution of Legendre's differential equation,

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+n(n+1)y=0$$ **(07 Marks)**

c. State Rodrigue's formula for Legendre polynomials and obtain the expression for $P_4(x)$ from it. Verify the property of Legendre polynomials in respect of $P_4(x)$ and also find $\int_{-1}^{1}x^3P_4(x)dx$. **(07 Marks)**

6 a. Two fair dice are rolled. If the sum of the numbers obtained is 4, find the probability that the numbers obtained on both the dice are even. **(06 Marks)**

b. Given that $P(\overline{A}\cap\overline{B})=\frac{7}{12}$, $P(A\cap\overline{B})=\frac{1}{6}=P(\overline{A}\cap B)$. Prove that A and B are neither independent nor mutually disjoint. Also compute P(A/B) + P(B/A) and $P(\overline{A}/\overline{B})+P(\overline{B}/\overline{A})$. **(07 Marks)**

c. Three machines M_1, M_2 and M_3 produces identical items. Of their respective outputs 5%, 4% and 3% of items are faulty. On a certain day, M_1 has produced 25% of the total output, M_2 has produced 30% and M_3 the remainder. An item selected at random is found to be faulty. What are the chances that it was produced by the machine with the highest output? **(07 Marks)**

7 a. In a quiz contest of answering 'Yes' or 'No', what is the probability of guessing atleast 6 answers correctly out of 10 questions asked? Also find the probability of the same if there are 4 options for a correct answer. **(07 Marks)**

b. Define exponential distribution and obtain the mean and standard deviation of the exponential distribution. **(07 Marks)**

c. If X is a normal variate with mean 30 and standard deviation 5, find the probabilities that (i) $26\le X\le 40$, (ii) $X\ge 45$, (iii) $|X-30|>5$. [Give that $\phi(0.8)=0.2881$, $\phi(2.0)=0.4772$, $\phi(3.0)=0.4987$, $\phi(1.0)=0.3413$] **(06 Marks)**

8 a. Certain tubes manufactured by a company have mean life time of 800 hrs and standard deviation of 60 hrs. Find the probability that a random sample of 16 tubes taken from the group will have a mean life time (i) between 790 hrs and 810 hrs, (ii) less than 785 hrs, (iii) more than 820 hrs. [$\phi(0.67)=0.2486$, $\phi(1)=0.3413$, $\phi(1.33)=0.4082$]. **(06 Marks)**

b. A set of five similar coins is tossed 320 times and the result is:

No. of heads:	0	1	2	3	4	5
Frequency:	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution. [Given that $\psi^2_{0.05}(5)=11.07$] **(07 Marks)**

c. It is required to test whether the proportion of smokers among students is less than that among the lectures. Among 60 randomly picked students, 2 were smokers. Among 17 randomly picked lecturers, 5 were smokers. What would be your conclusion? **(07 Marks)**

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10CS42

Fourth Semester B.E. Degree Examination, June / July 2014

Graph Theory & Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Let G be a simple graph of order n. If the size of G is 56 and the size of $\overline{G}$ is 80, what is n? **(05 Marks)**

b. Define isomorphism of graphs. Show that the following two graphs in Fig. Q1 (b) are isomorphic. **(05 Marks)**

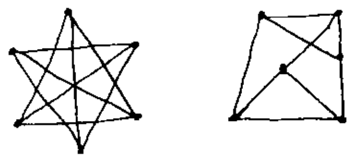

Fig. Q1 (b)

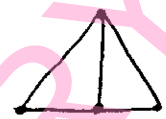

Fig. Q3 (b)

c. Define connected graph. Give an example of a connected graph G, where removing any edge e results in an disconnected graph. **(05 Marks)**

d. Discuss KÖnigsberg bridge problem. **(05 Marks)**

2 a. Define Hamilton cycle. If G = (V, E) is a loop free undirected graph with $|V| = n \geq 3$ and if $|E| \geq \binom{n-1}{2} + 2$, then prove that G has a Hamilton cycle. **(06 Marks)**

b. Define planar graph. If a connected planar graph G has n vertices, e edges and r regions, then prove that $n - e + r = 2$. **(07 Marks)**

c. Define chromatic number. Find the chromatic polynomial for the cycle of length 4. Hence find its chromatic number. **(07 Marks)**

3 a. Define a tree. Prove that in a tree T = (V, E), $|V| = |E| + 1$. **(06 Marks)**

b. Define : i) prefix code ii) balanced tree. Give one example for each. Find all the spanning trees of the graph as shown in Fig. Q3 (b). **(07 Marks)**

c. Construct an optimal prefix code for the letters of the word ENGINEERING. Hence deduce the code for this word. **(07 Marks)**

4 a. Define : i) Edge-connectivity ii) Vertex-connectivity and iii) Complete matching. Give an example for each. **(06 Marks)**

b. State Kruskal's algorithm. Apply Kruskal's algorithm to find a minimal spanning tree for the following weighted graph as shown in Fig. Q4 (b). **(07 Marks)**

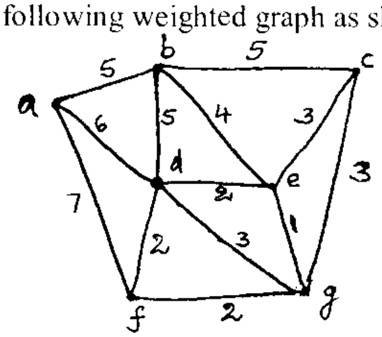

Fig. Q4 (b)

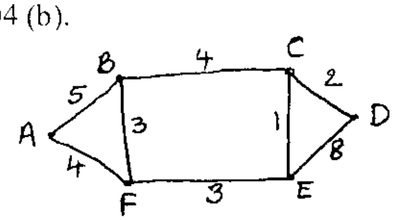

Fig. Q4 (c)

c. State Max-flow and Min-cut theorem. For the network shown below in Fig. Q4 (c), find the capacities of all the cut sets between the vertices A and D, and hence find the maximum flow. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

PART – B

5 a. How many arrangements are there for all letters in the word SOCIOLOGICAL? In how many of these arrangements: i) A and G are adjacent ii) all the vowels are adjacent. **(05 Marks)**

b. In how many ways can one distribute eight identical balls into four distinct containers so that,
i) no container is left empty. ii) the fourth container gets an odd number of balls. **(05 Marks)**

c. Determine the coefficient of $x^2y^2z^3$ in the expansion of $(3x-2y-4z)^7$. **(05 Marks)**

d. Using the moves R: $(x, y) \rightarrow (x+1, y)$ and U : $(x, y) \rightarrow (x, y+1)$ find in how many ways can one go,
i) From (0, 0) to (6, 6) and not rise above the line $y = x$.
ii) From (2, 1) to (7, 6) and not rise above the line $y = x - 1$.
iii) From (3, 8) to (10, 15) and not rise above the line $y = x + 5$. **(05 Marks)**

6 a. Determine the number of positive integers n such that $1 \le n \le 100$ and n is not divisible by 2, 3 or 5. **(06 Marks)**

b. Define derangement. There are eight letters to eight different people to be placed in eight different addressed envelopes. Find the number of ways of doing this so that at least one letter gets to the right person. **(07 Marks)**

c. Find the rook polynomial for the 3×3 board using the expansion formula. **(07 Marks)**

7 a. Find the generating functions for the following sequences:
i) $0^2, 1^2, 2^2, 3^2, \ldots\ldots$
ii) 0, 2, 6, 12, 20, 30, 42, ……. **(06 Marks)**

b. Find the number of ways of forming a committee of 9 students drawn from 3 different classes so that students from the same class do not have an absolute majority in the committee. **(07 Marks)**

c. Using exponential generating function, find the number of ways in which four of the letters in the word ENGINE be arranged. **(07 Marks)**

8 a. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(06 Marks)**

b. Solve the recurrence relation:
$a_{n+2} + 3a_{n+1} + 2a_n = 3^n$, $n \ge 0$ given $a_0 = 0, a_1 = 1$. **(07 Marks)**

c. Find the generating function for the recurrence relation, $C_n = 3C_{n-1} - 2C_{n-2}$ for $n \ge 2$, given $C_1 = 5, C_2 = 3$. Hence solve it. **(07 Marks)**

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Fourth Semester B.E. Degree Examination, June/July 2014
Design and Analysis of Algorithms

Time: 3 hrs.   Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Define three asymptotic notations. **(06 Marks)**

b. Design a recursive algorithm for solving tower of Hanoi problem and give the general plan of analyzing that algorithm. Show that the time complexity of tower of Hanoi algorithm is exponential in nature. **(08 Marks)**

c. With algorithm and a suitable example, explain how the brute force string matching algorithm works. Analyse for its complexity. **(06 Marks)**

2 a. Give general divide and conquer recurrence with necessary explanation. Solve the recurrence $T(n) = 2T(n/2) + 1$

$T(n) = T(n/2) + n$ **(06 Marks)**

b. Explain with suitable example a sorting algorithm that uses divide and conquer technique which divides the problem size by considering position. Give the corresponding algorithms and analyse for time complexity. **(08 Marks)**

c. Give the problem definition of a defective chessboard? Explain clearly how divide and conquer method can be applied to solve a 4×4 defective chess board problem. **(06 Marks)**

3 a. What is job sequencing with deadline problem? Find solution generated by job sequencing problem with deadlines for 7 jobs given profits 3, 5, 20, 18, 1, 6, 30 and deadlines 1, 3, 4, 3, 2, 1, 2 respectively. **(06 Marks)**

b. Define minimum cost spanning tree. Give high level description of Prim's algorithm to find minimum spanning tree and find minimum spanning tree for graph shown in Fig.Q3(b) using Prim's algorithm. **(08 Marks)**

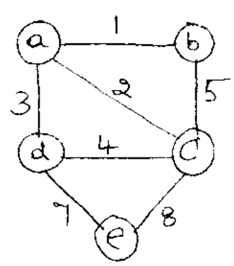

Fig.Q3(b)

c. What is a knapsack problem? Obtain solution for the knapsack problem using greedy method for n = 3, capacity m = 20 values 25, 24, 15 and weights 18, 15, 10 respectively. **(06 Marks)**

4 a. What is dynamic programming? Explain how would you solve all pair shortest paths problem using dynamic programming. **(06 Marks)**

b. Give the necessary recurrence relation used to solve 0/1 knapsack problem using dynamic programming. Apply it to solve the following instance and show the results n = 4, m = 5 values 12, 10, 20, 15 and weights are 2, 1, 3, 2 respectively. **(08 Marks)**

c. Solve the following TSP which is represented as a graph shown in Fig.Q4(c) using dynamic programming. **(06 Marks)**

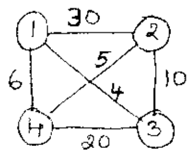

Fig.Q4(c)

PART – B

5 a. Explain the working of depth-first search algorithm for the graph shown in Fig.Q5(a).

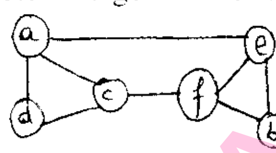

Fig.Q5(a) **(06 Marks)**

b. With pseudocode, explain how the searching for a pattern BARBER in the given text JIM_SAW_ME_IN_BARBER_SHOP is performed using Horspool's algorithm. **(08 Marks)**

c. With a suitable example, explain topological sorting. **(06 Marks)**

6 a. What is a decision tree? Give a decision tree for three element selection sort for arranging three items in ascending order. Give its asymptotic behavior. **(06 Marks)**

b. What are NP-complete problems and NP-hard problems? Apply four iterations of Newton's method to compute $\sqrt{2}$ and estimate the absolute and relative errors of the computations. **(08 Marks)**

c. What do you mean by lower bound algorithm? What are the advantages of finding the lower bound and give different methods of obtaining the lower bound? **(06 Marks)**

7 a. With necessary state space diagram, explain the solving of four-queen's problem by backtracking. **(10 Marks)**

b. For the given n×n cost matrix C for a job assignment problem find optimal solution using branch and bound method. Give complete state-space tree for the instance of assignment problem solved with best first branch and bound algorithm.

$$C = \begin{array}{c} \begin{array}{cccc} job1 & job2 & job3 & job4 \end{array} \\ \begin{bmatrix} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{bmatrix} \end{array} \begin{array}{l} \text{person a} \\ \text{person b} \\ \text{person c} \\ \text{person d} \end{array}$$

(10 Marks)

8 a. Explain different types of computational models. **(10 Marks)**

b. For given input 5, 12, 8, 6, 3, 9, 11, 12, 1, 5, 6, 7, 10, 4, 3, 5 to the prefix computation problem and ⊕ stands for addition, solve problem using work optimal algorithm. **(10 Marks)**

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Fourth Semester B.E. Degree Examination, June/July 2014
UNIX and Shell Programming

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. With a neat diagram, explain the architecture of UNIX operating system. List the features also. **(08 Marks)**
b. Explain the parent–child relationship of UNIX file system with a diagram. **(06 Marks)**
c. Explain with examples :
i) Absolute pathname and relative pathname
ii) Internal and external commands. **(06 Marks)**

2 a. Interpret the significance of seven fields of ℓs – ℓ output. **(07 Marks)**
b. Briefly explain the different ways of setting file permissions. **(07 Marks)**
c. With a diagram, explain 3 modes of Vi editor. **(06 Marks)**

3 a. What are wild cards? Explain the shells wild cards, with examples. **(08 Marks)**
b. What is a process? Explain the process creation mechanism? Why directory change can't be made in separate process. **(08 Marks)**
c. Explain the following environment variables, with examples :
i) HOME ii) PATH iii) IFS iv) SHELL. **(04 Marks)**

4 a. What are hard links and soft link? Explain with examples. **(06 Marks)**
b. Write a short note on find command. **(06 Marks)**
c. Explain the following filters with examples :
i) head iii) tail iii) cut. **(08 Marks)**

PART – B

5 a. Explain grep command with all options. **(10 Marks)**
b. What is sed? With example, explain line addressing and context addressing. **(10 Marks)**

6 a. What is shell programming? Write a shell script to create a menu which displays :
i) List of files ii) Contents of a file iii) Process status
iv) Current date v) Clear the screen vi) Current users of system. **(10 Marks)**
b. Explain shell features of 'for'. With syntax and examples. **(10 Marks)**

7 a. What is an awk? Explain all the built in variables used by awk. **(10 Marks)**
b. With syntax and examples, discuss the control flow statements used by awk. **(10 Marks)**

8 a. Write a Perl script to demonstrate the use of chop function. **(06 Marks)**
b. Write a Perl script to find the square root of command line arguments. **(06 Marks)**
c. Explain the string handling functions of Perl with appropriate examples. **(08 Marks)**

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Fourth Semester B.E. Degree Examination, June/July 2014
Microprocessor

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Define a microprocessor. Explain in detail the evolution of microprocessor in microprocessor age from 4004 MP to core-2 system. **(06 Marks)**

b. Explain in details with a neat figure the working of the internal architecture of the 8086MP. **(08 Marks)**

c. Explain in detail the various bits of a flag register for 8086 MP. **(06 Marks)**

2 a. Explain with an example why and how a 20 bit address is generated in 8086. **(05 Marks)**

b. Explain any five addressing modes in detail with examples that are supported in 8086 MP. **(10 Marks)**

c. Explain the concepts of protected mode of memory addressing. **(05 Marks)**

3 a. Write 8086 ALP to add 10 non-negative data items using string instructions. **(06 Marks)**

b. Explain the following instructions with examples:
i) CMP ii) LAMF iii) XCHG iv)LEA v) PUSH AX vi) LDS DI, [3000h]. **(06 Marks)**

c. Explain with examples the following assembler directives (any four):
i) ORG ii) DQ iii) PROC and ENDP iv) TYPE v) EVEN. **(08 Marks)**

4 a. Explain the various string manipulation instructions with examples. **(06 Marks)**

b. Explain the following instructions with examples any four:
i) DAA ii) MUL iii) ADC iv) SHR v) RCL. **(08 Marks)**

c. Explain the different types of jumps and cell instructions of 8086. **(06 Marks)**

PART – B

5 a. Write an assembly language program using C/C++ to perform the operation x + y = z with proper comments. **(10 Marks)**

b. Define modular programming. Using the concept of public and extra directives write a program which reads data in a program in one module which is then used by another module. **(06 Marks)**

c. Differentiate between macros and procedures. **(04 Marks)**

6 a. Describe in detail the use of the following signals:
i) ACE ii) RESET iii) NMI iv) HOLD v) $MN/\overline{MX}$ vi) QSI and QSQ. **(06 Marks)**

b. Explain in detail with a neat figure demultiplexing of address and data lines in 8086. **(06 Marks)**

c. Explain with a neat figure the working of 8086 in MIN mode configuration. **(08 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

7 a. Differentiate between memory mapped I/O and I/O mapped I/O. (04 Marks)
b. Design an 8086 based system to interface with i) 64K byte EPROM; ii) 64K byte RAM. Assume RAM is connected at 30000h and EPROM at F0000h. (08 Marks)
c. Explain how a 3-8 line decoder could be used to interface eight 8K memory chips. (08 Marks)

8 a. Explain different signals of 8255 PP and control words. (08 Marks)
b. Explain with a neat diagram the interfacing of stepper motor to 8086 using 8255 in detail. (06 Marks)
c. Explain the working of different blocks of 8254 PIT with a neat figure. (06 Marks)

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10CS46

Fourth Semester B.E. Degree Examination, June/July 2014

Computer Organization

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Draw the connection between processor and memory and mention the functions of each component in the connection. **(08 Marks)**

b. Write the difference between RISC and CISC processors. **(04 Marks)**

c. A program contain 1000 instructions. Out of that 25% instructions requires 4 clock cycles, 40% instructions requires 5 clock cycles and remaining requires 3 clock cycles for execution. Find the total time required to execute the program running in a 1 GHz machine. **(05 Marks)**

d. Add +5 and –9 using 2's compliment method. **(03 Marks)**

2 a. Explain immediate, indirect and indexed addressing modes. **(08 Marks)**

b. Explain different rotate instructions. **(06 Marks)**

c. Write ALP program to copy 'N' numbers from array 'A' to array 'B' using indirect addresses. (Assume A and B are the starting memory location of a array). **(06 Marks)**

3 a. Explain the following terms :
i) Interrupt service routine ii) interrupt latency iii) interrupt disabling. **(06 Marks)**

b. With a diagram, explain daisy chaining technique. **(06 Marks)**

c. What you mean by bus arbitration? Briefly explain different bus arbitration techniques. **(08 Marks)**

4 a. With a block diagram, explain how the printer is interfaced to processor. **(08 Marks)**

b. Explain the architecture and addressing scheme of USB. **(08 Marks)**

c. Define two types of SCSI controller. **(04 Marks)**

PART – B

5 a. Explain direct memory mapping technique. **(06 Marks)**

b. What is virtual memory? With a diagram, explain how virtual memory address is translated. **(08 Marks)**

c. Explain the working of 16 megabyte DRAM chip configured as 1 M × 16 memory chip. **(06 Marks)**

6 a. Design 4 bit carry look ahead logic and explain how it is faster them 4 bit ripple adder. **(08 Marks)**

b. Multiply 14 × – 8 using Booth's algorithm. **(06 Marks)**

c. Explain normalization, excess – exponent and special values with respect to IEEE floating point representation. **(06 Marks)**

7 a. With a diagram, explain typical single bus processor data path. **(08 Marks)**

b. Explain with neat diagram, the basic organization if a microprogrammed control unit. **(08 Marks)**

c. Differentials hardwired and microprogrammed control unit. **(04 Marks)**

8 a. Define and discuss Amdahl's law. **(06 Marks)**

b. With a diagram, explain a shared memory multiprocessor architecture. **(08 Marks)**

c. What is hardware multithreading? Explain different approaches in hardware multithreading. **(06 Marks)**

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MATDIP401

Fourth Semester B.E. Degree Examination, June / July 2014
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Define direction cosine and direction ratio of a line. Hence show that $l^2 + m^2 + n^2 = 1$. **(06 Marks)**

b. For any cube show that angle between any two diagonals is $\cos^{-1}\left(\frac{1}{3}\right)$. **(07 Marks)**

c. Define plane. Derive equation of plane in general form. **(07 Marks)**

2 a. Find equation of plane passing through A(-1, 1, 1), B(1, -1, 1) and perpendicular to plane $x + 2y + 2z - 5 = 0$ **(06 Marks)**

b. Show that the line $\frac{x-4}{2} = \frac{y-2}{3} = \frac{z-3}{10}$ is parallel to plane $2x + 2y - z = 6$. Find distance between them. **(07 Marks)**

c. Show that lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ are coplanar. Find point of intersection. **(07 Marks)**

3 a. Find sine and cosine of angle between the vectors $4i + 3j + k$, $2i - j + 2k$. **(06 Marks)**

b. Show that points (4, 5, -1), (0, -1, -1), (3, 9, 4), (-4, 4, 4) are coplanar using vector method. **(07 Marks)**

c. Prove that $\left[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}\right] = 2\left[\vec{a}, \vec{b}, \vec{c}\right]$. **(07 Marks)**

4 a. A particle moves along the curve $x = t^3+1$, $y = t^2$, $z = 2t + 5$. Find components of its velocity and acceleration at $t = 1$ in the direction $i + j + 3k$ **(06 Marks)**

b. Find directional derivative of $x^2 + y^2 + 4xyz$ at (1, -2, 2) in the direction $2i - 2j + k$. **(07 Marks)**

c. Show that $\text{grad}\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^2}$. **(07 Marks)**

5 a. For any scalar function ϕ show that $\text{curl}(\text{grad}\phi) = 0$. **(06 Marks)**

b. If $\vec{F} = \text{grad}\phi$, $\phi = x^2+y^2+z^2+xyz$, find $\nabla \cdot \left(\vec{F}\right)$ and $\nabla \times \left(\vec{F}\right)$ at (1, 1, 1). **(07 Marks)**

c. Find a, b, c so that $\vec{F} = (x + y + az)i + (x + cy + 2z)j + (x + 2y - z)k$ is irrotational. Find scalar function. **(07 Marks)**

6 a. Find Laplace Transform if t^n and hence find $L\left(t^{\frac{1}{2}}\right)$. **(06 Marks)**

b. Find $L\left[e^{2t}\cos 3t + e^{-t}\sin 2t + t\sin t\right]$. **(07 Marks)**

c. Find $L\left[\dfrac{e^{t}(\cos 3t - \cos t)}{t}\right]$. **(07 Marks)**

7 a. Find $L[\sin t\ \sin 2t\ \sin 3t]$. **(06 Marks)**

b. Find $L[f(t)]$ where $f(t) = \begin{cases} 1 & 0 < t \le 1 \\ t & 1 < t \le 2 \\ t^2 & t > 2 \end{cases}$. **(07 Marks)**

c. Find $L^{-1}\left\{\log\sqrt{\dfrac{s+a}{s-b}}\right\}$. **(07 Marks)**

8 a. Find $L^{-1}\left\{\dfrac{2s^2 - 6s + 5}{s^3 - 6s^2 + 11s - 6}\right\}$. **(10 Marks)**

b. Solve by Laplace transformation, $\dfrac{d^2y}{dt^2} + 7\dfrac{dy}{dt} + 10y = 4e^{-3t}$, given $y(0) = 0, y'(0) = -1$.

(10 Marks)

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